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Damping parameter design optimization in structural systems using an explicit H_∞ norm bound ☆

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Abstract

This paper examines the damping parameter design optimization problem for structural systems with collocated measurements and inputs so that H_∞ norm bound constraints are satisfied. We utilize a particular solution of the Bounded Real Lemma that provides an explicit upper bound on the H_∞ norm of a collocated structural system. Using this upper bound result the damping design is formulated as a linear matrix inequality (LMI) optimization problem with respect to the damping coefficients of the structural system. The formulation is particularly useful for large-scale structural systems where existing methods are computationally prohibitive. Numerical examples demonstrate the benefits and computational advantages of the proposed damping parameter design method.

1. Introduction

As an important component of passive structural systems design, the problem of damping parameters design has been studied for over a century [1]. The use of dashpots and tuned mass dampers (TMD) is two widely used methods of passive vibration attenuation. These simple devices have been proven very effective for reducing severe vibrations of machinery, buildings, bridges and many other mechanical and civil engineering systems with relatively low cost [2,3]. Such passive vibration control devices are often favored compared to active and semi-active controllers in many practical applications because of their reduced complexity including the absence of external driving power [4].

There are numerous optimization-based methods that have been examined for optimal placement and optimal value assignment of damping devices. To this end, optimality conditions have been derived and various parametric studies and gradient or steepest-descent algorithms have been proposed to minimize resonant amplitude, critical excitation response or system energy dissipation requirements [5–9]. However,

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these methods suffer from increased complexity and computational cost that could easily become prohibitive, especially for large-scale structural systems.

Recently, structural parameter design techniques have been proposed that consider the structural parameter selection and optimization of passive mechanical components as a control design problem. This approach allows the use of rigorous system theoretic and system gain mathematical tools to quantify system performance and to provide a connection to closed-loop response in controlled structural systems. Thus, optimal and robust control techniques, such as the H_2/LQR (linear quadratic regulator) and H_∞ optimization approaches, have been proposed for structural parameter design. The H_2 optimal dynamic output feedback control synthesis problems is connected to the solution of the standard LQG (linear quadratic Gaussian) problem, which is a combination of state estimation and state feedback control that can be solved using the corresponding Riccati equations. The state-space H_∞ norm control method based on the Riccati equation approach or the linear matrix inequality (LMI) formulation are now well-developed control synthesis tools. The optimal static state feedback and full-order dynamic output feedback H_∞ control synthesis problems can be solved using iterations on the corresponding Riccati solutions or via the computational solution of a convex LMI optimization problem [10–13]. On the other hand, the static output feedback and the fixed-order dynamic output feedback control synthesis problems are difficult computational problems since they require the solution of (nonconvex) bilinear matrix inequalities or linear matrix inequalities with coupling rank constraints [14,15]. Unfortunately, the use of such control-oriented methods for structural parameter design leads to a static output feedback formulation resulting in complex numerically cumbersome optimization problems [16–21].

Having strain or displacement information as the system output, feedback of the measured output rate has been proven to be very effective in adding local damping to the structure. The control of structural systems with collocated sensors and actuators has been shown to provide great advantages from a stability, passivity, robustness and an implementation viewpoint [22]. For example, collocated control can easily be achieved in a space structure when an attitude rate sensor is placed at the same location as a torque actuator [22,23]. Collocation of sensors and actuators leads to symmetric transfer functions. Several other classes of engineering systems, such as circuit systems, chemical reactors and power networks, can be modelled as systems with symmetric transfer functions. Stabilization, robustness, model reduction and control of such systems have been examined recently [24,25].

In this work, we use recently developed control-oriented algebraic tools to formulate the damping parameter design problem in collocated structural systems as an efficient convex LMI optimization problem [26]. By exploiting the particular structure of collocated structural systems, explicit upper bounds for the H_∞ norm of the system can be obtained. To this end, particular solutions of the Bounded Real Lemma (BRL) are used and an explicit expression for an upper bound of the H_∞ norm of such external symmetric system are obtained that require only the computation of the maximum eigenvalue of a symmetric matrix. Subsequently, the damping parameter design problem is formulated as a LMI optimization problem of minimizing the H_∞ norm bound with respect to the unknown damping coefficients.

LMI optimization problems have received increased attention recently in systems, controls and structural design applications. They constitute convex optimization problems that can be solved efficiently in polynomial time using interior point methods. Hence, global optimality of the corresponding solution is guaranteed. A collection of system analysis and control design problems that can be formulated in a LMI form can be found in Refs. [13,26].

The standard notation $>$ ($<$) is used in this paper to denote the positive (negative) definite ordering of symmetric matrices. The ith eigenvalue of a real symmetric matrix $\mathbf{N}$ will be denoted by $\lambda_i(\mathbf{N})$ where the ordering of the eigenvalues is defined as $\lambda_{\max}(\mathbf{N}) = \lambda_1(\mathbf{N}) \geqslant \lambda_2(\mathbf{N}) \geqslant \cdots \geqslant \lambda_n(\mathbf{N})$. The maximum singular value of a (not necessarily square) matrix $\mathbf{N}$ will be denoted by $\sigma_{\max}(\mathbf{N})$, which is also its spectral norm $\|\mathbf{N}\|$.

2. Analytical H_∞-norm bound for collocated systems

Consider the following vector second-order representation of a structural system with collocated velocity measurements and inputs:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}\mathbf{u}, \qquad \mathbf{y} = \mathbf{F}^{\mathrm{T}}\dot{\mathbf{q}}, \tag{1}$$

where $\mathbf{q}(t) \in \mathbb{R}^n$ is the generalized coordinate vector, $\mathbf{u}(t) \in \mathbb{R}^m$ is the input or disturbance vector and $\mathbf{y}(t) \in \mathbb{R}^k$ is the measured output vector. The matrices $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are symmetric positive-definite matrices that represent the structural system mass, damping and stiffness distribution, respectively.

The system has a state-space realization as follows:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}, \qquad \mathbf{y} = \mathbf{Cx} \tag{2}$$

with state-space matrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{F} \end{bmatrix},$$

$$\mathbf{C} = [\mathbf{0} \;\; \mathbf{F}^{\mathrm{T}}], \tag{3}$$

where the state vector is $\mathbf{x} = [\mathbf{q}^{\mathrm{T}} \;\; \dot{\mathbf{q}}^{\mathrm{T}}]^{\mathrm{T}}$. The transfer function $\mathbf{G}(s)$ of the system in Eqs. (2) and (3) is obtained as

$$\mathbf{G}(s) = s\mathbf{F}^{\mathrm{T}}(\mathbf{M}s^2 + \mathbf{D}s + \mathbf{K})^{-1}\mathbf{F}.$$

Notice that this transfer function is symmetric, i.e., $\mathbf{G}(s) = \mathbf{G}^{\mathrm{T}}(s)$. Hence, the system in Eqs. (2) and (3) is an *externally symmetric* state-space realization, that is, there exists a nonsingular matrix $\mathbf{T}$ such that

$$\mathbf{A}^{\mathrm{T}}\mathbf{T} = \mathbf{TA}, \qquad \mathbf{C}^{\mathrm{T}} = \mathbf{TB}. \tag{4}$$

This class of systems is more general than the class of *internally* or *state-space symmetric* systems that they satisfy the symmetry conditions (Eq. (4)) with a positive-definite transformation matrix $\mathbf{T}$ [27]. Obviously, state-space symmetry implies external symmetry, but the converse is not true, that is, there exist symmetric transfer matrices for which there is no internally symmetric realization. An analytical solution of the H_∞ control problem for internally symmetric systems has been presented in Ref. [28].

The H_∞ norm of a system is the peak magnitude of its frequency response function (FRF). In a time-domain interpretation, the H_∞ norm corresponds to the energy (or L_2 norm) gain of the system from the input $\mathbf{u}$ to the output $\mathbf{y}$ [13]. Hence, in this setting the H_∞ norm defines a disturbance rejection property of the system.

Recall that the H_∞ norm of the system in Eq. (2) is given by

$$\|\mathbf{G}\|_\infty = \sup_{\omega \in \mathbb{R}} \sigma_{\max}\{\mathbf{G}(\mathrm{j}\omega)\}, \tag{5}$$

where $\mathbf{G}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$ is the transfer function of the system and $\sigma_{\max}$ denotes the maximum singular value of a matrix. It is well known that for a stable linear time-invariant system, its H_∞ norm can be approximated iteratively, for example using a bisection method [29]. The following result shows that for a vector second-order realization described in Eqs. (2) and (3), an upped bound on its H_∞ norm can be computed using a simple explicit formula [30].

Theorem 1. *Consider the vector second-order system realization in Eqs.* (2) *and* (3). *The system has an* H_∞ *norm* γ *that satisfies*

$$\gamma < \bar{\gamma} = \lambda_{\max}(\mathbf{F}^{\mathrm{T}}\mathbf{D}^{-1}\mathbf{F}). \tag{6}$$

To prove this result recall the BRL characterization of the H_∞ norm of a system.

Lemma 1 (*Skelton et al. [13]*). *A stable system as in Eq.* (2) *has an* H_∞ *norm less than or equal to* γ *if and only if there exists a matrix* $\mathbf{P} \geqslant \mathbf{0}$ *satisfying*

$$\begin{bmatrix} \mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{PA} & \mathbf{PB} & \mathbf{C}^{\mathrm{T}} \\ \mathbf{B}^{\mathrm{T}}\mathbf{P} & -\gamma\mathbf{I} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} & -\gamma\mathbf{I} \end{bmatrix} \leqslant \mathbf{0}. \tag{7}$$

Recall also the following Schur complement formula [31].

Lemma 2. *The block matrix*

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{12}^{\mathrm{T}} & \mathbf{S}_{22} \end{bmatrix},$$

where $\mathbf{S}_{11}$ *and* $\mathbf{S}_{22}$ *are symmetric, is positive definite if and only if*

$$\mathbf{S}_{11} > \mathbf{0} \quad \textit{and} \quad \mathbf{S}_{22} - \mathbf{S}_{12}^{\mathrm{T}}\mathbf{S}_{11}^{-1}\mathbf{S}_{12} > \mathbf{0}$$

or

$$\mathbf{S}_{22} > \mathbf{0} \quad \textit{and} \quad \mathbf{S}_{11} - \mathbf{S}_{12}\mathbf{S}_{22}^{-1}\mathbf{S}_{12}^{\mathrm{T}} > \mathbf{0}.$$

Theorem 1 follows from the BRL condition and the following algebraic result [32].

Lemma 3. *Consider matrices* $\mathbf{\Gamma}$ *and* $\mathbf{Q}$ *such that* $\mathbf{\Gamma}$ *has full column rank and* $\mathbf{Q}$ *is symmetric positive definite. Then* $\mathbf{Q} \geqslant \mathbf{\Gamma}\mathbf{\Gamma}^{\mathrm{T}}$ *if and only if*

$$\lambda_{\max}(\mathbf{\Gamma}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{\Gamma}) \leqslant 1.$$

Proof of Theorem 1 (*Sketch*). The result follows from the BRL 1 by utilizing the following Lyapunov matrix:

$$\mathbf{P} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix}. \tag{8}$$

Application of the Schur complement formula in Lemma 2 results in the following condition:

$$-\mathbf{D} + \frac{1}{\gamma}\mathbf{F}\mathbf{F}^{\mathrm{T}} \leqslant \mathbf{0}. \tag{9}$$

Then, application of the Lemma 3 provides the bound in Eq. (6). □

Numerical examples in Ref. [30] demonstrate the validity and computational efficiency of the above analytical bound.

3. Damping design using the analytical bound approach

The analytical H_∞ norm upper bound of the collocated structural system in Eq. (1) given in Theorem 1 is solely dependent on the damping distribution matrix **D** and the input/output distribution matrix **F**. For lumped parameter systems, the damping metric **D** can be expressed in terms of the elemental damping coefficients as follows:

$$\mathbf{D} = \sum_{i=1}^{m} c_i \mathbf{T}_i, \tag{10}$$

where c_i denotes the viscous damping constant of the ith damper and $\mathbf{T}_i$ represents the distribution matrix of the corresponding damper in the structural system. The $\mathbf{T}_i$'s are given symmetric matrices with elements 0, 1 and -1 that define the structural connectivity of the damping elements in the structure. Then, using the Schur complement formula, the H_∞ norm upper bound condition in Eq. (6) can be re-written as

$$\begin{bmatrix} \sum_{i=1}^{m} c_i \mathbf{T}_i & \mathbf{F} \\ \mathbf{F}^{\mathrm{T}} & \gamma \mathbf{I} \end{bmatrix} \geqslant \mathbf{0}. \tag{11}$$

Practical structural system design specifications impose upper bound constraints on the values of the damping coefficients, that is

$$0 \leqslant c_i \leqslant c_{\max}. \tag{12}$$

Also, often an upper bound on the total available damping resources is imposed, that is

$$\sum_{i=1}^{m} c_i \leqslant c_{\text{total}}. \tag{13}$$

Based on the above discussion the damping design problem can be formulated as follows.

4. Damping design optimization problem

Consider the collocated structural system given in Eq. (1) with the damping distribution defined in Eq. (10). For a given positive scalar γ, the H_∞ norm of the system is less than γ if the following conditions with respect to the damping coefficients c_i are feasible:

$$\begin{bmatrix} \sum_{i=1}^{m} c_i \mathbf{T}_i & \mathbf{F} \\ \mathbf{F}^{\mathrm{T}} & \gamma \mathbf{I} \end{bmatrix} \geqslant \mathbf{0}, \tag{14}$$

$$0 \leqslant c_i \leqslant c_{\max}, \tag{15}$$

$$\sum_{i=1}^{m} c_i \leqslant c_{\text{total}}. \tag{16}$$

The above conditions constitute an LMI feasibility problem with respect to the damping coefficients c_i. Then, the optimization of damping coefficients can be achieved by solving the LMI optimization problem

$$\min_{c_i} \gamma \tag{17}$$

subject to the constraints defined in Eqs. (14)–(16). Recall that LMI optimization problems constitute convex optimization problems that can be solved effectively using recently developed interior point optimization algorithms [13,26].

Hence, the upper bound approach in the above LMI formulation provides a computationally efficient method to compute the damping coefficients of collocated structural systems. Since the assigned upper bound γ is always greater than or equal to the exact H_∞ norm of the system, the design result is conservative. However, our computational examples and experience with the proposed bound indicate that it indeed provides a good approximation of the exact H_∞ norm.

Remark. The significance and benefit of the proposed H_∞ norm upper bound damping parameter optimization approach is evident in the design of very large-scale structural systems where standard methods based on nonlinear optimization approaches are computationally prohibitive. The proposed LMI-based convex optimization formulation can address the design of structural systems with a large number of states and design variables.

5. Numerical examples

5.1. Single degree of freedom (1-dof) case

To demonstrate and motivate the above results consider the 1-dof case ($n = 1$) where $\mathbf{q}(t)$, $\mathbf{u}(t)$ and $\mathbf{y}(t)$ are scalar quantities in Eq. (1). For this scalar case, the magnitude of the FRF of the system in Eq. (1) is

$$|G(\mathrm{j}\omega)| = \frac{F^2|\omega|}{\sqrt{(K - M\omega^2)^2 + D^2\omega^2}}, \tag{18}$$

where M, D and K represent the scalar mass, damping and stiffness coefficients of the system. It can be easily observed that the magnitude of the FRF reaches its maximum at the natural frequency of this dynamic system, i.e., when

$$\omega = \omega_n = \sqrt{\frac{K}{M}}. \tag{19}$$

Thus, the above FRF magnitude satisfies the following bound at all frequencies:

$$|G(\mathrm{j}\omega)| = |G(\mathrm{j}\omega_n)| \leqslant \frac{F^2|\omega|}{D|\omega|} = \frac{F^2}{D}, \tag{20}$$

that is, $\|G\|_\infty \leqslant F^2/D$. This bound is precisely the one provided in Theorem 1 for the system. In fact, in this scalar case the above bound provides the *exact* H_∞ norm of the system, that is $\|G\|_\infty = F^2/D$. Therefore, the H_∞ norm bound $\|G\|_\infty \leqslant \gamma$ is achieved if and only if the damping coefficient D is selected to satisfy the bound

$$D \geqslant F^2/\gamma. \tag{21}$$

Notice that this result coincides with the bound obtained from Eq. (14).

As an example consider the case where $F = 10$, $M = 30\,\mathrm{kg}$ and $K = 500\,\mathrm{N/m}$. Then, for a desired H_∞ norm bound $\gamma = 0.5$ the designed value of the damping coefficient $D = 10^2/0.5 = 200\,\mathrm{N\,s/m}$. This result is confirmed from the Bode diagram of the system shown in Fig. 1 that obtains a maximum of $20\log_{10}(0.5) = -6.02\,\mathrm{dB}$ at $\omega = \sqrt{K/M} = 4.08\,\mathrm{rad/s}$.

5.2. Two-story shear building model

Now, let us consider a two-story shear building model with added viscous dampers as shown in Fig. 2. This model is considered in Ref. [5]. The two-degree-of-freedom system has masses $m_1 = m_2 = 100\,\mathrm{kg}$ and stiffness $k_1 = k_2 = 200\,\mathrm{N/m}$ relative to the base. The system is subject to the disturbance inputs u_1 and u_2 (such as wind gust disturbance on the structure). Our objective is to design the damping coefficients c_1 and c_2 that attenuate the velocities of the masses m_1 and m_2 caused by the disturbance forces u_1 and u_2. Specifically, we want to design the dampers c_1 and c_2 so that the system satisfies a given H_∞ norm performance constraint $\gamma \leqslant 0.5$ from the disturbance forces to the velocities v_1, v_2 of the masses m_1 and m_2.

The system has the collocated vector second-order form in Eq. (1) with structural matrices

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \qquad \mathbf{D} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix},$$

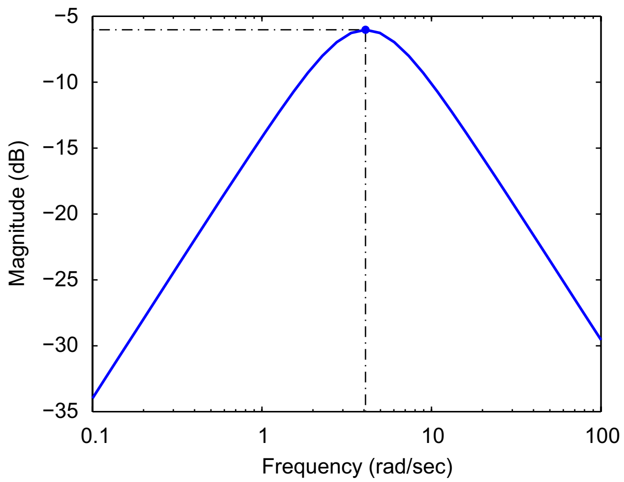

Fig. 1. Bode diagram of the one-degree-of-freedom example (magnitude in m/s).

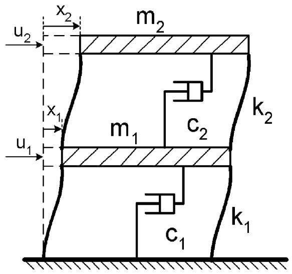

Fig. 2. Two-story shear building model with added viscous dampers.

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \quad \text{and} \quad \mathbf{F} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{22}$$

The design parameters are c_1 and c_2 and the damping distribution matrices $\mathbf{T}_1$ and $\mathbf{T}_2$ of the damping distribution matrix $\mathbf{D}$ are obtained from the expansion

$$\begin{aligned} \mathbf{D} &= \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \\ &= c_1 \mathbf{T}_1 + c_2 \mathbf{T}_2. \end{aligned} \tag{23}$$

In addition, the damping coefficients are subject to the constraints

$$c_i \leqslant 10\,\mathrm{N\,s/m}, \quad i = 1, 2. \tag{24}$$

Using the above parameters we solve the corresponding LMI feasibility problem described in Section 4. The corresponding damping coefficients are obtained as

$$c_1 = 5.04, \qquad c_2 = 6.49\,\mathrm{N\,s/m}, \tag{25}$$

and the exact H_∞ norm of the damped system is 0.481 which is slightly less than the desired bound 0.5. The frequency responses from the disturbance u_1 to output v_1, from u_2 to v_2, from u_1 to v_2 and from u_2 to v_1 are shown in Figs. 3 and 4. It can be seen that the designed system reduced the effect of the disturbances significantly and our design goal is achieved.

5.3. Five spring–mass–damper system example

Next, consider the five-degree-of-freedom mass, spring and damper interconnected system shown in Fig. 5. This example is borrowed and revised from Refs. [16,19].

Our design objective is to optimize the values of the damping coefficients c_i, $i = 1, \ldots, 5$ so that the H_∞ norm of the collocated system from the disturbance forces u_1 and u_2 to the velocities of masses m_2 and m_4 is minimized. The physical parameters of this system are selected as $m_i = 1\,\mathrm{kg}$, $k_i = 1\,\mathrm{N/m}$ and the damping coefficients c_i, $i = 1, \ldots, 5$ are the unknown design parameters.

The damping distribution matrix $\mathbf{D}$ of this system is given by

$$\mathbf{D} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 & 0 \\ 0 & -c_3 & c_3 + c_4 & -c_4 & 0 \\ 0 & 0 & -c_4 & c_4 + c_5 & -c_5 \\ 0 & 0 & 0 & -c_5 & c_5 \end{bmatrix} = \sum_{i=1}^{5} c_i \mathbf{T}_i. \tag{26}$$

For comparison and design trade-off purposes we consider a family of optimal damper designs using the results of the damping design optimization problem defined in Section 4. The obtained designs correspond to different values of the total damping capacity c_{total} ranging from 0.5 to 20 N s/m. The results of the optimal

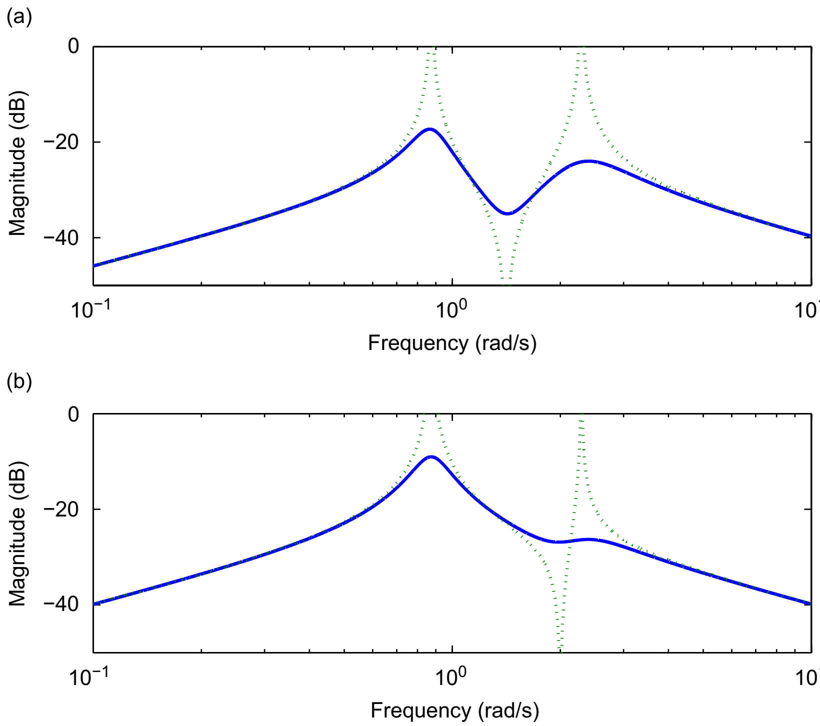

Fig. 3. Frequency responses of the undamped and damped system: (a) from $u1$ (N) to $v1$ (m/s) and (b) from $u2$ (N) to $v2$ (m/s). $\cdots$, Before damping design; —, after damping design.

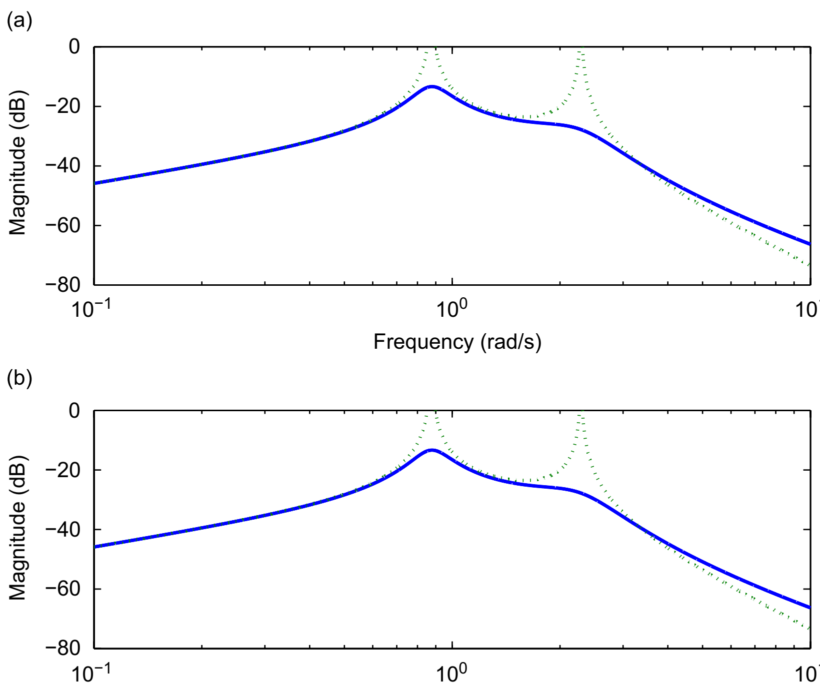

Fig. 4. Frequency responses of the undamped and damped system: (a) from $u1$ (N) to $v2$ (m/s) and (b) from $u2$ (N) to $v1$ (m/s). $\cdots$, Before damping design; —, after damping design.

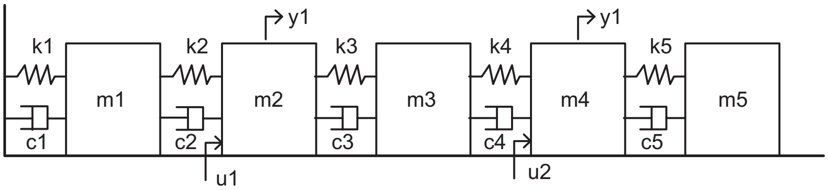

Fig. 5. Five mass–spring–damper system.

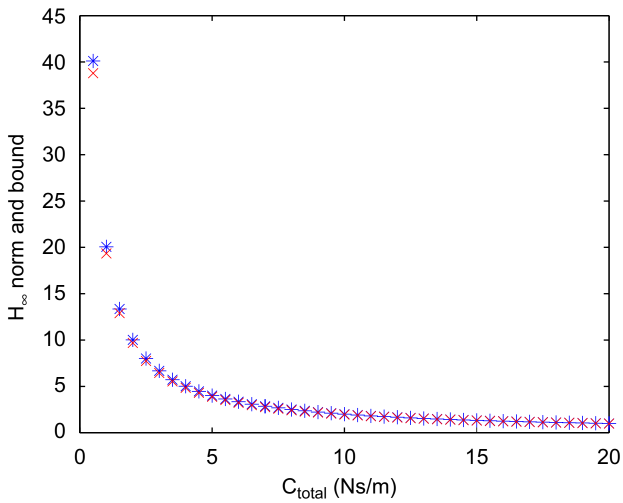

Fig. 6. Comparison of the H_∞ norm upper bound optimization result and the exact H_∞ norm of the designed systems: $*$, H_∞ norm upper bound optimization; $\times$, exact H_∞ norm.

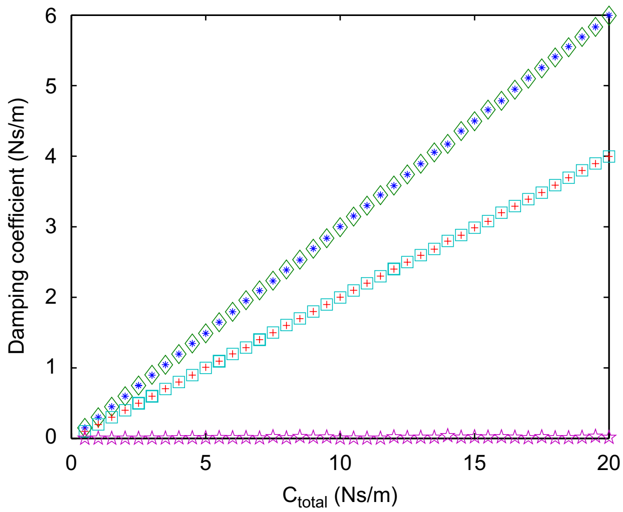

Fig. 7. Optimized damping coefficients using the H_∞ upper bound approach: $*$, c_1; $\diamond$, c_2; $+$, c_3; $\square$, c_4; $\star$, c_5.

designs using the H_∞ upper bound optimization approach are shown in Figs. 6 and 7. Fig. 6 shows the values of the H_∞ norm bound obtained using our upper bound optimization approach, as well as, the exact H_∞ norm that corresponds to each design as the total damping capacity c_{total} changes. Indeed, we observe that in each design the guaranteed H_∞ norm bound of the designed system and its actual H_∞ norm are very close. This result demonstrates that our upper bound optimization approach provides a good upper bound estimate of the actual H_∞ norm of the system. The values of the optimized damping parameters that correspond to each design are shown in Fig. 7. We observe that for each design $c_1 = c_2$ and $c_3 = c_4$ although c_5 is close to zero. This result is justified due to the location of the disturbance forces u_1 and u_2.

5.4. Application to a large-scale structural system

As a last example, we apply the proposed design method for the damping parameter design of a large-scale collocated structural system. We consider the finite element structural model for the assembly phase 8A-OBS of the International Space Station (ISS) shown in Fig. 8 with collocated measurements and inputs. This example follows the state-space model in Eq. (1) with 216 degrees-of-freedom [33]. We assume Rayleigh damping and we consider a damping parameter optimization (possibly implemented through active control means) to satisfy H_∞ norm specifications. Fig. 9 shows the H_∞ norm bound obtained by solving the optimization problem presented in Section 4 for different values of the total damping capacity c_{total} and the actual H_∞ norm of the structural system for each design. It is observed that the value of the H_∞ norm bound and the achievable H_∞ norm are extremely close. It should be noted that the use of traditional optimization methods for damping parameter design for this system could easily become prohibitive due to the high dimensionality of the system.

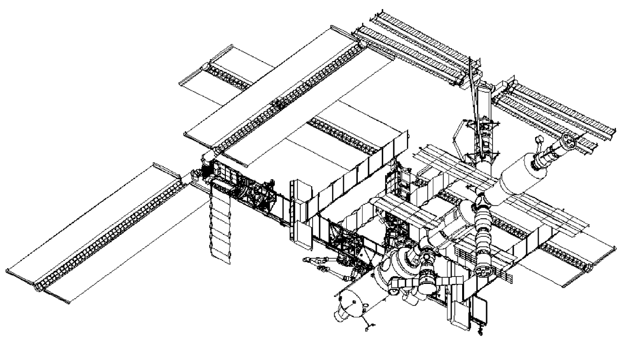

Fig. 8. The assembly phase 8A-OBS of the International Space Station.

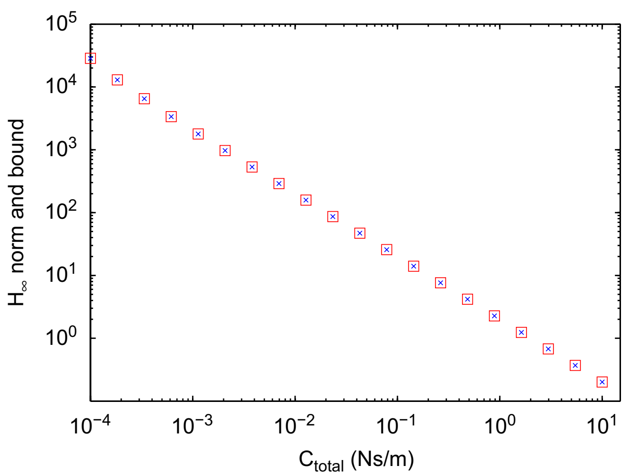

Fig. 9. Comparison of the H_∞ upper bound optimization result and the exact H_∞ norm of the ISS model: □, H_∞ norm upper bound optimization; ∗, exact H_∞ norm.

6. Conclusion

We have considered the damping parameter design optimization in structural systems with collocated measurements and inputs. By utilizing an explicit H_∞ norm bound, a computationally efficient linear matrix inequality formulation is proposed to design the damping parameters that guarantee a desired H_∞ norm of such structural systems. The approach takes into consideration constrains on the values of the damping parameters and the total available damping resources. Computational examples demonstrate the validity and effectiveness of the proposed H_∞ upper bound damping design approach. The design methods are applicable to multi-degree-of-freedom systems and are particularly useful for very large-scale systems where the existing damping coefficient design methods are computationally prohibitive.

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